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IMPROVED NUMERICAL SOLUTIONS FOR THE SIMULATION OF MICROWAVE CIRCUITS

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Abstract: The electromagnetic characteristics of microwave circuits can be described by the scattering matrix. This results in a three-dimensional boundary value problem, which can be solved using the Finite Difference method in the Frequency Domain (FDFD). A time consuming part of the FDFD-method is the solution of large systems of linear algebraic equations. The coeffcient matrix is sparse, symmetric, and indenite. Using multicoloring and independent set orderings essential numerical improvements are achieved.

1. INTRODUCTION

Maxwell's equations in a general source-free and linear medium in a time harmonic and integral form can be expressed as

$$\nabla \times \overline{H} = \frac{\partial D}{\partial t} \Rightarrow \oint_{\partial\Omega} \frac{1}{m_r m_0} \overline{B} d\overline{s} = \int_{\Omega} \left(jwe_0 e_r \overline{E} \right) d\overline{\Omega} \quad (1)$$

$$\nabla \times \overline{e} = -\frac{\partial \overline{B}}{\partial t} \Rightarrow \oint_{\partial\Omega} \overline{E} d\overline{s} = \int_{\Omega} \left(- j\overline{B} \right) d\overline{\Omega} \quad (2)$$

$$\nabla \cdot \overline{D} = 0 \Rightarrow \oint_{n\Omega} \left(e_r e_0 \overline{E} \right) d\overline{\Omega} = 0 \quad (3)$$

$$\nabla \cdot \overline{B} = 0 \Rightarrow \oint_{n\Omega} \overline{B} d\overline{\Omega} = 0 \quad (4)$$

Constitutive relations between the field quantities are determined by the macroscopic properties of the medium being considered.

$$\overline{D} = e\overline{E} \quad (5)$$

$$\overline{B} = m\overline{H} \quad (6)$$

We wish to solve Maxwell's equations on a regular three-dimensional domain ν that has a closed boundary surface denoted by ∂n . We assume that the domain ν has been discretized into structured orthogonal hexahedral cells. Each cell has a closed boundary surface denoted by $\nu\Omega$. Each face Ω is surrounded by a closed contour $\partial\Omega$. The linear isotropic material properties ε_r and μ_r are constant in each cell. In this paper, only lossless materials are taken into consideration. Therefore, the quantities ε_r and μ_r are real. The boundary conditions on ∂n are as follows: On some parts of the surface the tangential electric field is given by an eigenvalue problem (see [2] and [6]). These parts of ∂n are called ports. On the remaining parts of ∂n either the tangential electric field or the tangential magnetic field tan vanishes. In the former case the surface is called electric wall, and it describes a metal with infinite conductivity. The case is called magnetic wall. This concept is useful to describe a symmetry plane of the field (which allows to reduce the domain υ to one half of the volume of the structure under investigation).

The elementary cell of the Yee grid [13] is shown in Figure 1. The locations of the electric and magnetic fields do not coincide with the nodes i, j, k of the Cartesian grid. The electric field components are located at the centers of the edges of the cell and the magnetic flux density components are normal to the centers of the faces. The FDFD-method requires the use of a dual grid.

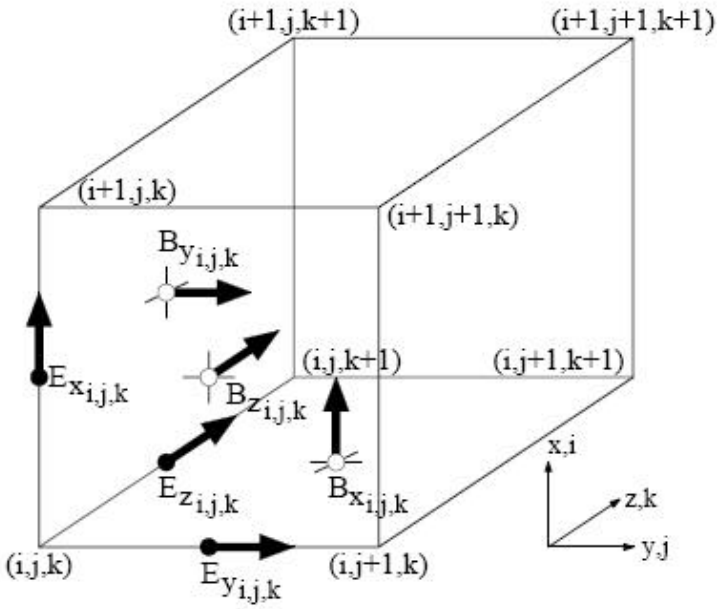

Figure 1: The elementary cell for the Yee algorithm with sampling locations of field quantities.

The dual grid and its structure are completely derivable from knowledge of the primary grid. Figure 2 shows an eight-cell hexahedral primary grid and its one interior dual cell. The primary grid is the grid that is initially created. The electric field components lie on the centers of the edges of the primary cell. We define the barycenters of all primary cells. Then we construct edges of the dual grid (dual edges) by connecting barycenters of adjacent cells with straight lines. The barycenters of two cells will be connected if and only if the two cells have a common face. Then the magnetic ux density components lie on the dual edges, i.e. on the edges of the dual cell.

2. THE SYSTEM OF ALGEBRAIC EQUATIONS

We use the lowest-order integration formulae:

$$\oint_{\partial\Omega} \bar{f} d\bar{s} \approx \sum_i f_i * (\pm s_i), \int_{\Omega} \bar{f} d\bar{\Omega} \approx f\Omega \qquad (7)$$

in order to approximate Maxwell's equations (see (1) – (4)). The closed path $\partial\Omega$ of the integration consists of straight lines of length si and is the path around the periphery of any cell face Ω of the primary and dual grid, respectively. The algebraic sign of length s_i in (7) depends on the direction of integration. f denotes the function value on the line with the length s. Ω is the area of any cell face of the primary and dual grid, respectively. The function value f lies on the cell face Ω.

Applying (7) to Equ. (2) yields (see [12])

The matrix A represents the curl operator in the Maxwellian equation (2) and consists only of the elements -1, 0 and 1. The diagonal matrices D_S and D_A contain the information on cell dimensions for the specified structure. Similarly, one has for Equ. (1)

Here, the matrix AT represents the curl operator in the Maxwellian equation (1) and is the transposed matrix of A. The diagonal matrices $D_{s/\mu}$ and $D_{A\varepsilon}$ contain the information on cell dimensions and material for the specified structure and the mesh.

Note that, strictly speaking, this approach represents a finite integration scheme. Nevertheless, since the resulting formulae are identical to the finite difference form, in most cases the more common term FDFD is used.

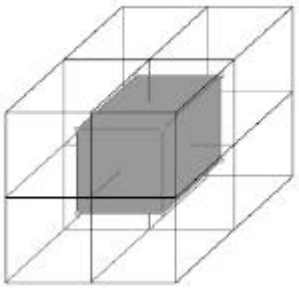

Figure 2: An eight cell primary grid and its one interior dual cell.

We combine the Equations by substituting the magnetic flux density components:

$$Q_1\bar{e} = 0; Q_1 = A^T D_{s/m} D_A^{-1} A S D_s - k_0^2 D_{Ae} \tag{8}$$

$$k_0 = w\sqrt{e_0 m_0} \tag{9}$$

In order to determine the scattering matrix, ports are defined on the surface of the structure. Their in-and outgoing wave modes act as sources for the field inside the domain (see [2] and [6]). Thus, a source term has to be induced by partitioning of the matrix Q1. The matrix representation of the Maxwellian equation (3) for the dual grid reads:

$$\oint_{n\Omega} \left(e_r e_0 \bar{E}\right) d\bar{\Omega} = 0 \Rightarrow B D_{Ae} \bar{e} = 0 \tag{10}$$

The matrix B represents the integral over a closed surface $\upsilon\Omega$ of the corresponding elementary cell and consists only of the elements -1, 0 and 1. We want to combine the information of Equations (8) and (10). Whereas (10) describes in the physical three-dimensional space a vectorial field, Equ. (10) represents a scalar one. Therefore, we build instead of (10) without loss of generality the vector equation:

$$e_r e_0 \nabla \left[\frac{1}{(e_r e_0)^2} \nabla e_r e_0 \bar{E} \right] = 0 \tag{11}$$

$$Q_2\bar{e} = 0, Q_2 = D_s^{-1} D_{Ae} B^T D_{V_{cc}}^{-1} B D_{Ae} \tag{12}$$

The diagonal matrix DV_{cc} contains the information on cell dimensions and material. We carry out a similar partitioning like (11) for the Equation (12) and then transform this equation into a symmetric one.

3. NUMERICAL EXAMPLE

The reduction of the computing time is demonstrated by calculating of the scattering matrix of a structure consisting of a microstrip line on a dielectric substrate grounded at one end by a via hole. The structure is divided into n_{xyz} = 60 984 elementary cells with n_x = 33, n_y = 28, and n_z = 66. The order of the system of linear algebraic equations is n = $3n_{xyz}$ = 182952. The total number of nonzeros of the matrix $\tilde{\tilde{A}} = \tilde{\tilde{A}}_0$ (26, 29) amounts to 1145738 for this example where only nnz = 664345 elements are stored. We apply an independent set ordering to the matrix A_0 with the level n_{lev} = 1 to obtain the reduced matrix A_1. The order of the reduced system of linear equations is n_1 = 84207. The total number of nonzeros of the matrix A_1 amounts to 1514129. The number of stored nonzeros is n_{nzr} = 799168.

We now consider four possibilities of preconditioning for the given coeficient matrix A to solve the linear system of equations.

1. The Eqn. is to be solved for n_{lev} = 0. This system of linear algebraic equations is used in [2].

2. Using the preconditioner $\hat{M} = \hat{M}_1 \hat{M}_2$ the system is to be solved for $n_{lev} = 0$.
3. Using the preconditioner $\hat{M} = \hat{M}_1 \hat{M}_2$ the system is to be solved for $n_{lev} = 1$.
4. The preconditioned system with the coeficient matrix A is to be solved for $n_{lev} = 1$ by using Eisenstat's trick.

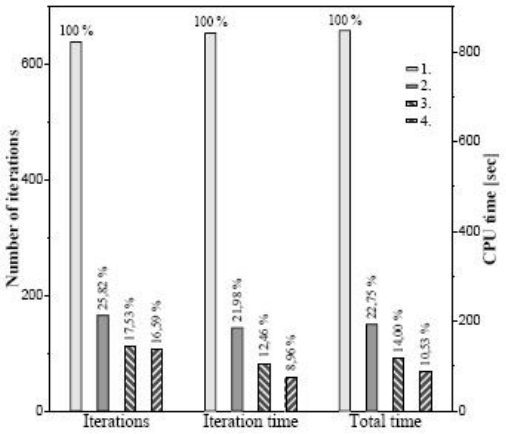

Figure 3: Number of iterations and execution times for the calculation of the scattering matrix.

Figure 3 shows the number of iterations and execution times (in seconds) for the four methods of preconditioning. The time data refer to a SGI workstation. The system of linear algebraic equations was solved by the algorithm described in [3]. The stopping criterion was in each case a reduction of the norm of the residual by 10^{-8}. Both the time for the iteration algorithm and the total time for the subroutine call are given. We observe that the SSOR preconditioner combined with the independent set ordering is very efective in solving the linear system of equations.

4. CONCLUSIONS

The Finite Difference method in Frequency Domain allows the computation of the scattering matrix of a given structure for a number of simultaneously excited modes. This is an advantage compared with calculations in the Time Domain. The price to be paid is the time-consuming solution of large systems of linear algebraic equations. We have studied various methods to do this effectively and found that the SSOR preconditioning combined with the independent set ordering is a very effcient method.

REFERENCES

[1] Barrett, R., Berry, M., Chan, T., Demmel, J., Donato, J., Dongarra, J., Eijkhout, V., Pozo, R., Romine, C., van der Vorst, H., Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods, SIAM, Philadelphia, PA, 1994;
[2] Beilenho_, K., Heinrich, W., Hartnagel, H. L., Improved Finite- Difference Formulation in Frequency Domain for Three-Dimensional Scattering Problems, IEEE Transactions on Microwave Theory and Techniques, Vol. 40, No. 3, March 1992;
[3] Freund, R. W., Nachtigal, N. M., A new Krylov-subspace method for symmetric indeffnite linear systems, AT&T Numerical Analysis Manuscript, Bell Labs, Murry Hill, NJ, 1994;
[4] Hebermehl, G., Schlundt, R., Zscheile, H., Heinrich, W., Improved Numerical Solutions for the Simulation of Monolithic Microwave Integrated Circuits, Preprint No. 236, Weierstras-Institut fur Angewandte Analysis und Stochastik im Forschungsverbund Berlin e.V., April 1996;
[5] Paige, C. C., Saunders, M. A., Solution of Sparse Indeffnite Systems of Linear Equations, SIAM J. Numer. Anal. 12, 4, pp. 617-629, September 1975
[6] Weiland, T., On the unique numerical solution of Maxwell eigenvalue problems in three dimensions, Particle Accelerators, Vol. 17, pp. 227-242, 1985.